Chapter 5

Simplify.

1. $\left(5x^2+3x-7\right)-\left(3x^2-x+4\right)$

SOLUTION:

$$(5x^2+3x-7)-(3x^2-x+4)$$
$$=5x^2-3x^2+3x+x-7-4$$
$$=2x^2+4x-11$$

ANSWER:

$2x^2+4x-11$

2. $\left(2y^2-4y-5\right)+\left(3y^2-2y+1\right)$

SOLUTION:

$$(2y^2-4y-5)+(3y^2-2y+1)$$
$$=2y^2+3y^2-4y-2y-5+1$$
$$=5y^2-6y-4$$

ANSWER:

$5y^2-6y-4$

3. $2cd(3c-4d)+4d(c+2d)$

SOLUTION:

Distribute.

$$2cd(3c-4d)+4d(c+2d)$$
$$=6c^2d-8cd^2+4cd+8d^2$$

There are no like terms, so the polynomial is simplified.

ANSWER:

$6c^2d-8cd^2+4cd+8d^2$

4. $(r-s)(r+s)(3r-2s)$

SOLUTION:

$$(r-s)(r+s)(3r-2s)$$
$$=(r^2-rs+rs-s^2)(3r-2s)$$
$$=(r^2-s^2)(3r-2s)$$
$$=3r^3-2r^2s-3rs^2+2s^3$$

ANSWER:

$3r^3-2r^2s-3rs^2+2s^3$

Simplify.

5. $\frac{4x^3y^4-10xy^5}{2xy}$

SOLUTION:

$$\begin{aligned}\frac{4x^3y^4-10xy^5}{2xy} &= \frac{4x^3y^4}{2xy}-\frac{10xy^5}{2xy}\\ &= \frac{4}{2}x^{3-1}y^{4-1}-\frac{10}{2}x^{1-1}y^{5-1}\\ &= 2x^2y^3-5y^4\end{aligned}$$

ANSWER:

$2x^2y^3-5y^4$

6. $\left(x^3+4x^2-4x-7\right)\div(x+1)$

SOLUTION:

$$\begin{array}{r} x^2+3x-7 \\ x+1\overline{)x^3+4x^2-4x-7} \\ \underline{(-)(x^3+x^2)} \\ 3x^2-4x \\ \underline{(-)(3x^2+3x)} \\ -7x-7 \\ \underline{(-)(-7x-7)} \\ 0 \end{array}$$

ANSWER:

x^2+3x-7

Find $p(-2)$ and $p(4)$ for each function.

8. $p(x) = -2x^2+5x-3$

SOLUTION:

$$\begin{aligned}p(x) &= -2x^2+5x-3\\ p(-2) &= -2(-2)^2+5(-2)-3\\ &= -2(4)-10-3\\ &= -8-13\\ &= -21\end{aligned}$$

$$\begin{aligned}p(x) &= -2x^2+5x-3\\ p(4) &= -2(4)^2+5(4)-3\\ &= -2(16)+20-3\\ &= -32+17\\ &= -15\end{aligned}$$

ANSWER:

–21, –15

9. $p(x) = -3x^3 - x^2 + 2x - 8$

SOLUTION:

$$\begin{aligned} p(x) &= -3x^3 - x^2 + 2x - 8 \\ p(-2) &= -3(-2)^3 - (-2)^2 + 2(-2) - 8 \\ &= -3(-8) - 4 - 4 - 8 \\ &= 24 - 16 \\ &= 8 \end{aligned}$$

$$\begin{aligned} p(x) &= -3x^3 - x^2 + 2x - 8 \\ p(4) &= -3(4)^3 - (4)^2 + 2(4) - 8 \\ &= -3(64) - 16 + 8 - 8 \\ &= -192 - 16 \\ &= -208 \end{aligned}$$

ANSWER:

8, –208

10. $p(x) = x^4 - 5x^3 + 4x + 6$

SOLUTION:

$$\begin{aligned} p(x) &= x^4 - 5x^3 + 4x + 6 \\ p(-2) &= (-2)^4 - 5(-2)^3 + 4(-2) + 6 \\ &= 16 - 5(-8) - 8 + 6 \\ &= 16 + 40 - 2 \\ &= 54 \end{aligned}$$

$$\begin{aligned} p(x) &= x^4 - 5x^3 + 4x + 6 \\ p(4) &= (4)^4 - 5(4)^3 + 4(4) + 6 \\ &= 256 - 5(64) + 16 + 6 \\ &= 256 - 320 + 22 \\ &= -42 \end{aligned}$$

ANSWER:

54, –42

Chapter 5

Factor completely. If the polynomial is not factorable, write *prime*.

12. $6x^4 - 5y^7$

SOLUTION:

$6x^4$ and $5y^7$ are not perfect cubes or perfect squares.

There are also no common factors between these two terms.

Therefore, the polynomial is prime.

ANSWER:
prime

13. $27a^3 - 125d^3$

SOLUTION:

$$\begin{aligned} 27a^3 - 125d^3 &= (3a)^3 - (5d)^3 \\ &= (3a - 5d)[(3a)^2 + (3a)(5d) + (5d)^2] \\ &= (3a - 5d)(9a^2 + 15ad + 25d^2) \end{aligned}$$

ANSWER:
$(3a - 5d)(9a^2 + 15ad + 25d^2)$

14. $2ax^2 - 3bx^2 + cx^2 - 2ay^2 + 3by^2 - cy^2$

SOLUTION:

$$\begin{aligned} &2ax^2 - 3bx^2 + cx^2 - 2ay^2 + 3by^2 - cy^2 \\ &= 2ax^2 - 2ay^2 - 3bx^2 + 3by^2 + cx^2 - cy^2 \\ &= 2a(x^2 - y^2) - 3b(x^2 - y^2) + c(x^2 - y^2) \\ &= (x^2 - y^2)(2a - 3b + c) \\ &= (x + y)(x - y)(2a - 3b + c) \end{aligned}$$

ANSWER:
$(x - y)(x + y)(2a - 3b + c)$

Given a polynomial and one of its factors, find the remaining factors of the polynomial.

15. $x^3 - 2x^2 - 5x + 6$; $x - 1$

SOLUTION:
Use synthetic division.

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & |0 \end{array}$$

Factor the remaining quadratic.

$$x^2 - x - 6 = (x-3)(x+2)$$

ANSWER:
$(x+2)(x-3)$

16. $2x^3 - x^2 - 25x - 12$; $x + 3$

SOLUTION:
Use synthetic division.

$$\begin{array}{r|rrrr} -3 & 2 & -1 & -25 & -12 \\ & & -6 & 21 & 12 \\ \hline & 2 & -7 & -4 & |0 \end{array}$$

Factor the remaining quadratic.

$$\begin{aligned} 2x^2 - 7x - 4 &= 2x^2 - 8x + x - 4 \\ &= 2x(x-4) + 1(x-4) \\ &= (2x+1)(x-4) \end{aligned}$$

ANSWER:
$(2x+1)(x-4)$

Find all the zeros of each function.

18. $f(x) = x^3 - 4x^2 - 7x + 10$

SOLUTION:
First, determine the total number of zeros.

$$f(x) = x^3 - 4x^2 - 7x + 10$$

There are 2 sign changes for the coefficients of $f(x)$, so the function has 0 or 2 positive real zeros.

$$\begin{aligned} f(-x) &= (-x)^3 - 4(-x)^2 - 7(-x) + 10 \\ &= -x^3 - 4x^2 + 7x + 10 \end{aligned}$$

There is 1 sign change for the coefficients of $f(-x)$, so $f(x)$ has 1 negative real zero.

Thus, $f(x)$ has 3 real zeros or 1 real zero and 2 imaginary zeros.

The possible real zeros are ±1, ±2, ±5, and ±10.

Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrr} 1 & 1 & -4 & -7 & 10 \\ & & 1 & -3 & -10 \\ \hline & 1 & -3 & -10 & |0 \end{array}$$

Factor the remaining quadratic.

$$x^2 - 3x - 10 = (x - 5)(x + 2)$$

The function has zeros at 1, 5, and –2.

ANSWER:
–2, 1, 5

19. $f(x) = x^4 - 8x^2 - 9$

SOLUTION:
First, determine the total number of zeros.

$$f(x) = x^4 - 8x^2 - 9$$

There is one sign change for the coefficients of $f(x)$, so the function has 1 positive real zero.

$$\begin{aligned} f(-x) &= (-x)^4 - 8(-x)^2 - 9 \\ &= x^4 - 8x^2 - 9 \end{aligned}$$

There is 1 sign change for the coefficients of $f(-x)$, so $f(x)$ has 1 negative real zero.

Thus, $f(x)$ has 2 real zeros and 2 imaginary zeros.

The possible real zeros are ±1, ±3, and ±9.

Use synthetic substitution to evaluate $f(x)$ for real values of x.

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & -8 & 0 & -9 \\ & & 3 & 9 & 3 & 9 \\ \hline & 1 & 3 & 1 & 3 & |0 \end{array}$$

$$\begin{array}{r|rrrr} -3 & 1 & 3 & 1 & 3 \\ & & -3 & 0 & -3 \\ \hline & 1 & 0 & 1 & |0 \end{array}$$

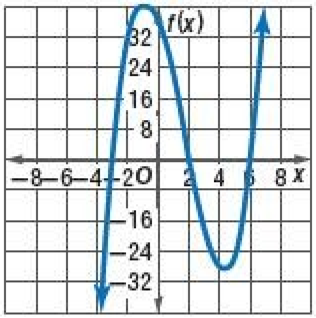
The remaining quadratic cannot be factored. Find the other zeros using the Quadratic formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{0 \pm \sqrt{(0)^2 - 4(1)(1)}}{2(1)} \\ &= \frac{\pm\sqrt{-4}}{2} \\ &= \frac{\pm 2i}{2} \\ &= \pm i \end{aligned}$$

The function has zeros at ***i***, –***i***, 3, and –3.

ANSWER:
–3, 3, ***i***, –***i***

25. Refer to the graph below. Find all the zeros of $f(x) = x^3 - 5x^2 - 12x + 36$.



SOLUTION:
The graph appears to cross the *x*-axis at –3, 2, and 6.

Test these zeros with synthetic division.

$$\begin{array}{r|rrrr} 2 & 1 & -5 & -12 & 36 \\ & & 2 & -6 & -36 \\ \hline & 1 & -3 & -18 & |0 \end{array}$$

Factor the remaining quadratic.

$$x^2 - 3x - 18 = (x-6)(x+3)$$

The zeros are 2, 6, and –3.

ANSWER:
–3, 2, 6